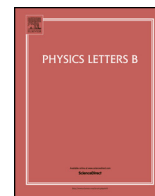




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Inflation, symmetry, and B-modes



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ABSTRACT

We examine the role of using symmetry and effective field theory in inflationary model building. We describe the standard formulation of starting with an approximate shift symmetry for a scalar field, and then introducing corrections systematically in order to maintain control over the inflationary potential. We find that this leads to models in good agreement with recent data. On the other hand, there are attempts in the literature to deviate from this paradigm by invoking other symmetries and corrections. In particular: in a suite of recent papers, several authors have made the claim that standard Einstein gravity with a cosmological constant and a massless scalar carries conformal symmetry. They claim this conformal symmetry is *hidden* when the action is written in the Einstein frame, and so has not been fully appreciated in the literature. They further claim that such a theory carries another hidden symmetry; a global $SO(1, 1)$ symmetry. By deforming around the global $SO(1, 1)$ symmetry, they are able to produce a range of inflationary models with asymptotically flat potentials, whose flatness is claimed to be protected by these symmetries. These models tend to give rise to B-modes with small amplitude. Here we explain that standard Einstein gravity does not in fact possess conformal symmetry. Instead these authors are merely introducing a *redundancy* into the description, not an actual conformal symmetry. Furthermore, we explain that the only real (global) symmetry in these models is not at all hidden, but is completely manifest when expressed in the Einstein frame; it is in fact the *shift symmetry* of a scalar field. When analyzed systematically as an effective field theory, deformations do not generally produce asymptotically flat potentials and small B-modes as suggested in these recent papers. Instead, deforming around the shift symmetry systematically, tends to produce models of inflation with B-modes of appreciable amplitude. Such simple models typically also produce the observed red spectral index, Gaussian fluctuations, etc. In short: simple models of inflation, organized by expanding around a shift symmetry, are in excellent agreement with recent data.

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1. Introduction

The theory of cosmological inflation [1,2], a phase of acceleration expansion in the early universe, is in good agreement with a range of observations. It is able to account for the large-scale homogeneity and isotropy of the universe, as well as providing a beautiful mechanism for the origin of large scale fluctuations. A missing component of the theory is a preferred model for the inflationary dynamics, although many interesting models have been proposed throughout the last few decades.

The simplest inflationary models involve Einstein gravity sourced by a scalar field ϕ and a potential $V(\phi)$. If we truncate the action at two derivatives, the action can be written, without loss of generality, as

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R + \frac{1}{2} (\partial\phi)^2 - V(\phi) \right] \quad (1)$$

where $M_{Pl} \equiv 1/\sqrt{8\pi G}$ and we are using the signature $+- - -$. If we choose the potential to simply be a cosmological constant, we would have a possibility of de Sitter space, though it would never end. So one normally imagines that the potential has some shape to it, including a minimum with $V \sim 0$, where inflation can end. The slow-roll conditions for a prolonged phase of inflation are

$$\epsilon \equiv \frac{M_{Pl}^2}{2} \left(\frac{V'}{V} \right)^2; \quad \epsilon \ll 1, \\ \eta \equiv M_{Pl}^2 \left(\frac{V''}{V} \right); \quad |\eta| \ll 1. \quad (2)$$

These conditions typically require the potential to be rather flat over a Planckian or super-Planckian domain in field space $\Delta\phi$.

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A generic potential $V(\phi)$ would not usually have this property. In fact many generic potentials that emerge in top-down models do not have this property. For example, if we parameterize the potential as a series expansion in powers of ϕ as follows (let's impose a $\phi \rightarrow -\phi$ symmetry for simplicity)

$$V(\phi) = \Lambda_0 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4 + \sum_{n=6}^{\infty} \frac{c_n}{M_{Pl}^{n-4}}\phi^n \quad (3)$$

Then if the coefficients are some fairly random numbers, and lets say $c_n \sim \mathcal{O}(1)$, then the required flatness of the potential is usually spoiled.

So to make progress one often invokes some type of symmetry structure. The most basic version of this is to imagine that ϕ carries a *shift symmetry* $\phi \rightarrow \phi + \phi_0$. This sets all the coefficients of the above potential to zero and obviously leaves a flat potential. But since this would be too strong, one then relaxes the shift symmetry slightly, i.e., allows a weak breaking of the shift symmetry by introducing very small values for the c_n , etc. This is said to be “technically natural” as the symmetry is restored in the limit in which the coefficients are set to zero. As we will describe later, this idea ultimately underpins the “chaotic inflation” model [3]. Related arguments occur for “natural inflation” in which one imagines that ϕ is a Goldstone boson associated with some spontaneously broken (global) symmetry [4]. This automatically forces the coefficients c_n to vanish. One then imagines that the underlying global symmetry is broken by some quantum effects, perhaps by non-perturbative effects as in the case of the axion, to generate small but non-zero coefficients. In other contexts, such as string theory, other possible structures emerge to control symmetries, such as “monodromies”, which can control the shape of the potential in an interesting way [5,6]. We will carefully study this general framework in Section 6.

Currently, we do not know if any of these symmetry arguments are on the right track, but they do organize the action into a sensible effective field theory and lead to interesting testable predictions. With fantastic precision in recent CMB observations [7–9], including polarization data, this program of model building is very worthwhile.

In this paper, we examine a recent claim of a new class of inflation models based on conformal symmetry. In Section 2 we describe these models. In Section 3 we review the meaning of conformal symmetry. In Section 4 we explain why this new class of models does not carry a physical conformal symmetry. In Section 5 we make the actual physical (global) symmetry in the models manifest and recognize it as a standard shift symmetry. In Section 6 we show how to deform around this standard shift symmetry within the framework of effective field theory. In Section 7 we discuss the consequences of various models for the amplitude of B-modes; contrasting those based on fine tuning and those based on symmetry. Finally, we discuss in Section 8.

2. New class of symmetry models?

Recently, a new class of inflation models organized by symmetry was put forward by several authors [10–17] (related ideas are also being examined in the context of bouncing cosmologies [18–20]). The basic new claim centers around the structure of standard Einstein gravity. It is claimed that standard Einstein gravity, even with a cosmological constant, carries a *conformal symmetry*. This is quite a dramatic claim, especially since such a model appears to carry two mass scales: the Planck mass M_{Pl} and the energy scale of the cosmological constant $\Lambda^{1/4}$. If we include a massless scalar field, the action is the following:

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R + \frac{1}{2}(\partial\phi)^2 - \Lambda \right] \quad (4)$$

So how could it possibly be that such a theory is actually conformal? The answer, they say, is that this conformal symmetry is *hidden* [10–17].

To exhibit this hidden conformal symmetry they introduce another scalar field χ which forms a doublet with the other scalar under a global $SO(1, 1)$ symmetry, as follows [17]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{12}(\chi^2 - \psi^2)R + \frac{1}{2}(\partial\psi)^2 - \frac{1}{2}(\partial\chi)^2 - \frac{\lambda}{4}(\psi^2 - \chi^2)^2 \right] \quad (5)$$

Notice that the kinetic term for χ is negative; which is a ghost term (however, in this context it does not lead to an instability). This action does not contain any explicit dimensionful parameters. The dramatic claim is that this action is *conformal* and it is connected to the above action. To claim this, the authors point out that this action is unchanged under the following transformations

$$\begin{aligned} g_{\mu\nu} &\rightarrow e^{2\alpha} g_{\mu\nu} \\ \psi &\rightarrow e^{-\alpha} \psi \\ \chi &\rightarrow e^{-\alpha} \chi \end{aligned} \quad (6)$$

which they refer to as a “local conformal symmetry”. Since α is an arbitrary function we can use it to “gauge fix” the scalar fields in a way we choose. In particular we can gauge fix

$$\chi^2 - \psi^2 = 6M_{Pl}^2 \quad (7)$$

This condition can be parameterized by writing

$$\begin{aligned} \psi &= \sqrt{6} M_{Pl} \sinh(\phi/\sqrt{6} M_{Pl}) \\ \chi &= \sqrt{6} M_{Pl} \cosh(\phi/\sqrt{6} M_{Pl}) \end{aligned} \quad (8)$$

Then upon substitution into Eq. (5) we find the action given in Eq. (4) of standard Einstein gravity with a free massless scalar and a cosmological constant $\Lambda = 9\lambda M_{Pl}^4$. So it would appear as though standard Einstein gravity with a cosmological constant is actually conformally invariant, but that its conformal symmetry is hidden by gauge fixing.

The next step is to deform the symmetries in order to build interesting models for inflation. The procedure that has been advocated is to return to the action in Eq. (5) and keep the conformal symmetry intact (they say it is a local or gauge symmetry so it should not be broken), but they choose to break the global $SO(1, 1)$ symmetry in the following way [17]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{12}(\chi^2 - \psi^2)R + \frac{1}{2}(\partial\psi)^2 - \frac{1}{2}(\partial\chi)^2 - \frac{\lambda}{4}F(\psi/\chi)(\psi^2 - \chi^2)^2 \right] \quad (9)$$

where F is some dimensionless function of the ratio of ψ to χ . Notice that this action is unchanged under the transformations given in Eq. (6) although F breaks the global $SO(1, 1)$ symmetry (unless F is a constant). Then by gauge fixing to the Einstein frame, as before, we are led to the following gauge fixed action

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R + \frac{1}{2}(\partial\phi)^2 - V(\phi) \right] \quad (10)$$

with

$$V(\phi) = \Lambda F(\tanh(\phi/\sqrt{6}M_{Pl})) \quad (11)$$

(again with $\Lambda = 9\lambda M_{Pl}^4$). This has the nice property that for many choices of F , such as $F(x) \propto x^n$, this potential $V(\phi)$ asymptotes to a constant at large (super-Planckian) field values. Since it asymptotes to a constant for super-Planckian field values then we can expect slow-roll inflation to occur at such values. Indeed, the slow-roll conditions $\epsilon \ll 1$ and $|\eta| \ll 1$ will be readily satisfied for many choices of F . So it is quite impressive by simply appealing to some symmetries, in particular a conformal symmetry and a deformed global symmetry, one can build many models of slow-roll inflation with asymptotically flat potentials. One also finds that these models generally predict [14]

$$n_s \approx 1 - \frac{2}{N_e}, \quad r \approx \frac{12}{N_e^2} \quad (12)$$

where n_s is the scalar spectral index, r is the tensor-to-scalar ratio, and N_e is the number of e-foldings of inflation (usually $50 \lesssim N_e \lesssim 60$). We will discuss these predictions further in Section 7.

In the rest of this note, we show that while these are some beautiful ideas, the above analysis hides some important subtleties. In particular, (i) by carefully defining conformal symmetry, we show that these models *do not* actually carry conformal symmetry, and (ii) by deforming around the global symmetry in the sense of effective field theory, we show that these models do not generically yield asymptotically flat potentials. We also comment on some other interesting attempts in the literature to obtain a conformal theory of gravitation.

3. What conformal symmetry is

Let us begin by defining conformal symmetry in the context of field theory. The first ingredients we need are some matter degrees of freedom ψ_i , and some dynamics governed by a Lagrangian \mathcal{L} . Let's allow for some non-trivial metric $g_{\mu\nu}$ that is treated as a *background*. The action is

$$S = \int d^4x \sqrt{-g} \mathcal{L}(\psi_i, \partial_\mu \psi_i) \quad (13)$$

The idea is to ask the following question: Does the action change if we perform a conformal change to the metric? That is, if we consider a background metric $g_{\mu\nu}$ and then rescale it as follows

$$g_{\mu\nu} \rightarrow \Omega(x)^2 g_{\mu\nu} \quad (14)$$

we wish to know if the dynamics is different in this new metric. Notice that the idea is to really change the actual metric, not simply our representation of the metric, i.e., we wish to explore different space-times, not a mere rewriting of a given same space-time.

We may also allow the ψ_i to transform with some power of Ω as

$$\psi_i \rightarrow \Omega(x)^{\Delta_i} \psi_i \quad (15)$$

where Δ_i is known as the “scaling dimension” of ψ_i . If for some choice of Δ_i the action returns to itself, then we obviously have a symmetry, a so-called “conformal symmetry”. In this special circumstance the physics is unchanged for different choices of conformally related metrics.

Some simple examples include pure electromagnetism, $\mathcal{N} = 4$ super-Yang Mills, and massless $\lambda\phi^4$ theory with non-minimal coupling to the background Ricci scalar $-\phi^2 R/12$. The first two of these examples are exact at the quantum level, while the third

example is only true classically. One consequence of the conformal symmetry is that the trace of the stress-energy tensor vanishes. Notice that it obviously requires a very special form for the Lagrangian for this conformal symmetry to exist. For instance, the Lagrangian obviously cannot possess any explicitly dimensionful parameters, such as mass terms, as this would immediately violate scale invariance (which is a necessary condition for conformal invariance).

4. What conformal symmetry is not

4.1. Dynamical space-time

In the previous section we defined a conformal symmetry for some matter degrees of freedom with respect to some *background* metric. Could it be possible that a conformal symmetry can extend to the case of a dynamical metric? Indeed, the claim of these authors is that the action given in Eq. (9) is conformally invariant when treating *both* the scalar (matter) fields as dynamical and the metric itself as dynamical.

Indeed, it is true that for the action given in Eq. (5), it is unchanged after performing the transformation of Eqs. (14) and (15) with $\Delta_i = -1$ for the pair of scalar fields; this was earlier described in Eq. (6) with $\Omega = e^\alpha$. However, there is a very important difference between the case of a background metric and a dynamical metric. In the case of a background metric the transformation in Eq. (14) changes the actual metric. However in the case of a dynamical metric this transformation is actually just a *field redefinition*. This does not change the actual metric, but only the *representation* of the metric. This is actually true for any gauge transformation; they leave the fields/states *invariant*, by definition.

Hence the transformations reported earlier in Eq. (6) are merely gauge transformations and not an actually changing of the metric. This is associated with the fact that there is a *redundant* degree of freedom in the action. This redundancy can be eliminated by gauge fixing. We did this earlier; we cut down from two scalar fields to one, by gauge fixing to the so-called Einstein frame.

Real symmetries are precisely those that remain *after* gauge fixing.¹ In this case it is simple to see that the theory does not have a conformal symmetry, since the Einstein frame gauge fixed action shows that there exist explicit mass scales that break scale (and conformal) symmetry; namely the Planck mass M_{Pl} and the energy scale of the cosmological constant $\Lambda^{1/4}$. Furthermore, it is relatively straightforward to see that there are loop corrections that generate a tower of higher dimension (derivative) operators, suppressed by the Planck scale. This evidently breaks conformal symmetry. Also, if we examine the deformed action expressed in the Einstein frame (see Eqs. (10), (11)) the existence of the potential shows that conformal symmetry is broken. For instance, a $\lambda\phi^4$ term carries a conformal anomaly, etc. Furthermore, for a potential of the form $V \sim \tanh(\phi/\sqrt{6}M_{Pl})$, we can Taylor expand it around $\phi = 0$, and see that it is evidently a tower of operators which, even at the classical level, break conformal symmetry.

Instead for a theory of gravitation to carry conformal symmetry, when gravity is treated dynamically, requires some very special structure; a point we will return to in Section 8.1.

¹ In some cases, the symmetries can be hidden after gauge fixing. For example, the Higgs mechanism can hide internal (global) symmetries when we gauge fix in the unitary gauge. However, even in this case, the symmetry is still manifest in some sectors of the theory and the global symmetry can still be checked to be present by the identification of a conserved quantity by the Noether theorem. In the models studied here, there are no sectors of the theory that carry the purported conformal symmetry, nor any conserved quantities. On the other hand, there is a real global $SO(1, 1)$ symmetry, which is, indeed, manifest after gauge fixing; we will return to this in Section 5.

4.2. Background space-time

To further drive home this point, let's turn to another case where it is extremely important to disentangle field redefinitions from actual field changes. This problem can even emerge when studying a fixed background space-time.

To begin, consider the following action of a single scalar field ϕ without dynamical gravity. We may in fact be simply interested in flat space, or conformally flat space, but let's include a metric to express the action in a generally co-ordinate invariant way

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4} \phi^4 - \sum_{n=6}^{\infty} \frac{c_n}{M^{n-4}} \phi^n \right] \quad (16)$$

For a range of reasons, one would not normally be tempted to suggest that this theory is conformally invariant. The background metric is taken to be non-dynamical; so that part is standard. However, the field carries a mass term, plus there are a tower of higher dimension operators suppressed by some mass scale M , the field does not carry the conformal coupling, and the trace of the stress-tensor is non-zero. Hence, we hope it is evident that this theory is not conformally invariant.

Nevertheless if one confuses *redundancies* for symmetries, then one might think that actually it does carry conformal symmetry. To make this point, let's continue in the spirit of the authors and introduce a pure gauge, or redundant, degree of freedom σ . We now consider the following action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} e^{2\sigma} \partial_\mu (e^{-\sigma} \phi) \partial_\nu (e^{-\sigma} \phi) - \frac{1}{2} e^{2\sigma} m^2 \phi^2 - \frac{\lambda}{4} \phi^4 - \sum_{n=6}^{\infty} \frac{c_n}{M^{n-4}} e^{-(n-4)\sigma} \phi^n \right] \quad (17)$$

This action is unchanged under the following set of gauge transformations

$$\begin{aligned} g_{\mu\nu} &\rightarrow e^{2\alpha} g_{\mu\nu} \\ \phi &\rightarrow e^{-\alpha} \phi \\ \sigma &\rightarrow \sigma - \alpha \end{aligned} \quad (18)$$

Hence, following the same reasoning that is used by these authors, one would conclude that even this theory carries a conformal symmetry. However, this is in fact nothing more than a field redefinition of ϕ , etc.; not an actual change in the field. We can (and should) gauge fix away this extra degree of freedom σ . We can gauge fix $\sigma = 0$ and then we recover the action in Eq. (16). Hence this theory of course does *not* carry conformal symmetry, even though it can be rewritten in a way that gives the impression that it does (for instance it is simple to check that the trace of the stress-tensor T^μ_μ that is derived from Eq. (17) is non-zero). We hope this makes it very clear that pure gauge versions of conformal symmetries are not real symmetries.

5. Global symmetries

While these models do not possess conformal symmetry, they do possess a global $SO(1,1)$ symmetry that relates ψ and χ . An attempt to deform this global symmetry is presented by the introduction of the function $F(\psi/\chi)$ in Eq. (9). However, it is unusual to deform a symmetry by introducing a function that depends on

a redundant degree of freedom. This inevitably means that the power counting that is being invoked is scrambled by the redundancy. Instead to make the symmetry and its deformations manifest, it is best to first remove this extra redundant degree of freedom by gauge fixing to the Einstein frame. With the symmetry in place this simplifies to Eq. (4) which carries a manifest global symmetry: a shift symmetry

$$\phi \rightarrow \phi + \phi_0 \quad (19)$$

Indeed, the Einstein frame is the frame that makes symmetries as manifest as possible. In the next section we examine this shift symmetry in a rigorous way.

6. Effective field theory

So, having gauge fixed to the Einstein frame, to make the symmetries manifest, we can begin deforming away from this shift symmetry. There are two basic ways to do this: (i) perturbatively, and (ii) non-perturbatively. In this section we will describe how to deform the symmetries in a systematic and controlled way, according to the principles of effective field theory.

Firstly, we note that the starting action that carries the shift-symmetry (Eq. (4)) is non-renormalizable. There will inevitably be an infinite tower of corrections to the action. However, the corrections that are generated perturbatively will respect the global shift symmetry. This means that the generated corrections will be *derivative corrections*. This means the full Lagrangian should include a tower of corrections of the form

$$\Delta \mathcal{L} = \sum_{n=2} \frac{d_n}{M^{4n-4}} (\partial\phi)^{2n} + \sum_{n=2} \frac{g_n}{M^{4n-4}} (M_{Pl}^2 R)^n + \dots \quad (20)$$

where the second term is shorthand for various possible contractions of the Riemann tensor. The dots indicate various other corrections involving higher derivative terms (box operator, etc.) and cross terms between derivative of ϕ and the Riemann curvature tensor. We cannot know what is the characteristic value of M , the mass scale that sets this expansion. It would be associated with heavy fields that we integrate out. But we can, as a model building assumption, take it to be very large, say, $M \sim M_{Pl}$. In this case we can safely ignore *all* these higher order derivative corrections. This is because the characteristic length scale during inflation is H^{-1} , which is several orders of magnitude longer than the Planck length, suppressing such higher derivative terms. This means that we can simply focus on the action in Eq. (4), under the assumption that M is sufficiently large, and consider how to deform the shift symmetry.

6.1. Perturbative corrections

Let us now consider adding corrections that break the shift symmetry, giving rise to a potential function $V(\phi)$. For example, the first natural terms to consider is a mass term and a possible quartic term for the classical potential

$$V_{cl}(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4} \phi^4 \quad (21)$$

In order for this model to give rise to the correct amplitude of scalar fluctuations, requires $m \lesssim 10^{13}$ GeV, $\lambda \lesssim 10^{-12}$. Having broken the shift symmetry, one should expect a tower of corrections to be generated at the quantum level. Indeed, graviton loops will generate such corrections of the form

$$\Delta V = \sum_{n=6} \frac{c_n}{M_{Pl}^{n-4}} \phi^n \quad (22)$$

However, it is very important to note the role of symmetry. Since the shift symmetry is restored in the $m, \lambda \rightarrow 0$ limit, then so too should these quantum general corrections. Indeed, at one loop, one finds that the quantum generated corrections to a classical potential take the form

$$\Delta V_{1-loop} = \left(a_1 \frac{V_{cl}''(\phi) V_{cl}(\phi)}{(4\pi)^2 M_{Pl}^2} + a_2 \frac{V_{cl}^2(\phi)}{(4\pi)^2 M_{Pl}^4} \right) \log(\phi) \quad (23)$$

where $a_{1,2} = \mathcal{O}(1)$ numbers that do not concern us here. Evaluating this for $m \ll M_{Pl}$, $\lambda \ll 1$, and $\phi \sim M_{Pl}$, we see that these corrections are negligibly small. Hence the classical potential in (21) is stable against perturbative quantum gravity corrections. One might be concerned that it is not technically natural for the mass to be small, but this is only a problem if the ϕ interactions are sufficiently large. So if we take the limit in which we ignore λ at the classical level. Then the residual potential

$$V_{cl}(\phi) = \frac{1}{2} m^2 \phi^2 \quad (24)$$

leaves a mass whose value is technically natural to be $m \ll M_{Pl}$ as there are no scalar–scalar interaction to drive it to large values. There are graviton corrections only, which are Planck suppressed, leading to reasonably small corrections to Δm^2 . Hence this classic model of inflation [3] is stable against perturbative quantum gravity corrections that arise in the effective field theory, and the mass itself is stable against radiative corrections. Hence, a consistent use of effective field theory around a shift symmetry leads to a candidate simple model for inflation.² Its cosmological predictions are

$$n_s \approx 1 - \frac{2}{N_e}, \quad r \approx \frac{8}{N_e} \quad (25)$$

which we will discuss further in Section 7.

Note this does not mean that this model will be readily attainable in a top-down approach. That is, it is non-trivial to obtain this low energy effective field theory from a microscopic theory. One needs to obtain the appropriate mass scale and the approximate shift symmetry to be respected to an excellent accuracy. The reason this is not trivial to achieve is that the field value ϕ is super-Planckian during inflation. By computing the evolution of the field during the course of inflation, it is simple to show

$$\Delta\phi \approx 2\sqrt{N_e} M_{Pl} \quad (26)$$

A microscopic theory may give rise to a large tower of Planck suppressed corrections even at the level of the classical effective potential. So although this low energy Lagrangian is radiatively stable, it is unclear if it will arise from a microscopic theory.

One way to potentially avoid the super-Planckian behavior of ϕ is to consider a large number of fields; this appears in the so-called “N-flation” models [23]. One can check that for a typical field ϕ_i , its typical displacement is (using the Pythagorean theorem)

$$\Delta\phi \approx 2\sqrt{N_e} M_{Pl} / \sqrt{N} \quad (27)$$

where N is the number of scalars. For N of a few hundred, this leads to sub-Planckian field values. This is helpful in gaining control over various higher order corrections that naturally emerge in top-down models. Although it is not clear if all corrections can be kept under control in string compactifications.

² Recently, in Refs. [21,22] it was found that one can go even further: by promoting ϕ to a complex field with an approximate $U(1)$ symmetry, one can also achieve baryogenesis in this simple model of inflation organized by symmetry.

6.2. Non-perturbative corrections

Another possible way to deform around the shift symmetry, is to note that all global symmetries are expected to be broken in quantum gravity. This does not necessarily imply a perturbative breaking, but a possible non-perturbative breaking of the shift symmetry. Or it may be broken by some other type of non-perturbative dynamics.

For definiteness, imagine that ϕ is a Goldstone boson associated with the spontaneous breaking of a global symmetry. In this case, the field is must be periodic. Lets call the symmetry breaking scale F , leading to a period $\phi_{period} = 2\pi F$. In this case the non-perturbative generated corrections must be a collection of harmonics of the form

$$V = V_0 + \sum_{n=1} V_n \cos(n\phi/F) \quad (28)$$

The coefficients V_n may be associated with some non-perturbative effect, such as instantons. In some cases, we can imagine that the leading harmonic is dominant. So lets approximate the potential as a single cosine. By setting aside the (late-time) cosmological constant, we write the potential as

$$V = \frac{V_0}{2} (1 + \cos(\phi/F)) \quad (29)$$

This is the so-called “natural inflation” model [4]. For details of the predictions for n_s and r , see Appendix A. We will discuss this further in Section 7. One finds that in order to achieve a nearly scale invariant spectrum, the parameter F must satisfy $F \gtrsim M_{Pl}$. This does not seem trivial to achieve, as it would indicate a super-Planckian symmetry breaking scale. A related direction is to imagine a field ϕ that moves in some “spiral” in field space, via a so-called “monodromy” [5,6]; these models also seem promising.

7. Consequences for B-modes

Here we examine the consequences for the amplitude of primordial B-modes that arise from the tensor modes generated during inflation. We will consider two different classes of large-field models: namely those built on a cancellation of terms that tend to appear in the “conformal” models and elsewhere in the literature, and those built on deforming around a shift symmetry. We will then also consider small field models.

7.1. Models based on fine tuning

There exist many large field models ($\Delta\phi \gtrsim M_{Pl}$) that rely upon the cancellation of a tower of terms in the potential. For instance, lets return to the “conformal symmetry” models described earlier in the paper (recall that they do not carry a real conformal symmetry, but only a redundancy). Recall that the potential in the Einstein frame took the form $V \sim F(\tanh(\phi/\sqrt{6} M_{Pl}))$. For some simple choices of the function F , this leads to models that at large field values take the form

$$V(\phi) \approx V_0 \left(1 - e^{-\sqrt{\frac{2}{3}} \frac{\phi}{M_{Pl}}} \right) \quad (30)$$

(we have absorbed a possible overall coefficient of the exponential into ϕ .) This leads to the tensor-to-scalar ratio, that we mentioned earlier, of $r \approx 12/N_e^2$. There are various types of models that tend to this exponentially flat behavior at large ϕ (including the original $R + R^2$ model [24], large non-minimal coupling models [25, 26], etc.). For $N_e \sim 55$, this leads to $r \approx 0.003$. This is consistent

with WMAP and Planck data [7,8], and will require significant improvement in technology to detect (including the identification of various foregrounds that can contaminate B-modes)

However, as we showed earlier, these models arise from not rigorously deforming around a manifest symmetry according to the principles of effective field theory. This can be seen here in this result for the potential $V(\phi)$. The potential is a tower of operators in powers of ϕ . This tower has the amazing property that the terms tend to cancel against one another at large ϕ , so as to produce an asymptotically flat V . Another way to see this is to introduce the $SO(1, 1)$ breaking term F in a different way, such as

$$V_J(\psi, \chi) = \frac{\lambda}{4}(\psi^2 - F(\psi/\chi)\chi^2)^2 \quad (31)$$

As long as $F(\psi/\chi) \neq 1$ for large ψ, χ , then this special flatness does not occur. For example, if we choose $F = 1.01$, this would appear to be some “small” breaking of the $SO(1, 1)$ symmetry, but it ruins the asymptotic flatness. Instead one is typically lead to completely different potentials in the Einstein frame.

So since the coefficients in the above (30) exponential for V are *not* determined by symmetry (recall that the underlying theory does not carry any conformal symmetry, and the global symmetry was scrambled when the action was formulated) this is a form of *fine tuning*. (This effects other models also [27,28].) The coefficients are chosen to reproduce this special function, even though there is no symmetry that actually organizes them into this form. One consequence of this very special choice of coefficients, leading to this very special exponentially flat potential, is that the amplitude of B-modes is small.³

7.2. Models based on symmetry

On the other hand, by expanding around a shift symmetry according to the principles of effective field theory, it is more common to produce potentials that continue to change at large field values, rather than flatten to a constant. As we mentioned earlier, if we introduce a mass term as the leading term that breaks the shift symmetry $V(\phi) = \frac{1}{2}m^2\phi^2 + \dots$, we will not generate large corrections within the effective field theory. Furthermore, this leads to a consistent large field model of inflation that does not rely upon a tower of operators whose coefficients conspire to cancel against one another. Instead, higher corrections, such as $\lambda\phi^4$, tend to steepen the potential.

Furthermore, if one has some knowledge of the microscopic theory, one might be led to other sorts of potentials. For example a periodic potential would naturally emerge for a Goldstone boson that arises from a symmetry that is broken by non-perturbative quantum effects. Other possibilities include fields whose shift symmetry is maintained, approximately, by a monodromy over large field ranges.

In these types of models, there is no general preference for the field to become asymptotically flat. Rather the symmetry may simply protect the potential to remain “sufficiently flat” over large field values for inflation to occur. Generally this leads to relatively large amplitude B-modes. For instance, in the $V \sim m^2\phi^2$ model, the prediction of $r \approx 8/N_e$ leads to $r \approx 0.15$. For monodromy models, the predictions are comparable, though a little smaller. For the case of the cosine potential, arising from non-perturbative quantum effects, the prediction is $r \leq 0.15$, depending on the ratio F/M_{Pl} . In general, these amplitudes for B-modes should be detectable in upcoming CMB experiments, although it is unclear if they are completely compatible with existing Planck data [8].

7.3. Small field models

Another possibility is to focus on small field models. In this case, a tower of corrections suppressed by the Planck scale seems less problematic. However, one should at least be concerned about the $\sim \phi^6/M_{Pl}^2$ term from spoiling the flatness of the potential. This is sometimes referred to the η -problem. This quintic piece can raise η , leading to only a small number of e-foldings of inflation. So in this case, one only needs to fine tune a single operator to be small, which seems more reasonable.

These models are constrained to produce negligible gravity waves, or B-modes in the CMB, by the “Lyth bound” [29]

$$r < 0.5 \left(\frac{\Delta\phi}{M_{Pl}} \right)^2 \quad (32)$$

So for reasonably large values of r , namely $r \gtrsim 0.1$, these small field models are not allowed as $\Delta\phi$ would need to be of the order of or greater than M_{Pl} . Such models would be ruled out by a discovery of B-modes.

8. Discussion

8.1. Could gravity be conformal?

Earlier we examined the claims in the literature that standard Einstein gravity with a cosmological constant is in fact a conformal field theory. We showed that in fact this theory does not carry conformal symmetry, instead authors were introducing only a redundancy into the description. However, it is interesting to examine whether some substantial modifications to standard Einstein gravity might actually result in a conformal theory.

One interesting possibility is that the Newton’s constant flows at high energies to a fixed point due to quantum corrections [30]. In addition, one would need all couplings to flow to a fixed point (and there would be infinitely many). In this case the theory would flow to a conformal field theory in the UV. This is interesting to pursue, but may be incompatible with the density of states of black holes [31]. Instead, the counting of states in the UV for black holes is comparable to the counting of states of a conformal field theory in one lower dimension. This is related to the famous AdS/CFT correspondence. Another possible way that gravity could be conformal is to consider Weyl gravity and its variants (although it is unclear if such theories can be made sensible).

8.2. Effective field theory and quantum gravity

We showed that a useful way to build simple models of inflation is to start with a shift symmetry for a scalar field and deform around it. From the effective field theory, this is a consistent approach as it leads to models that are radiatively stable; the perturbatively generated quantum corrections are small. We showed that simple models, including either perturbative or non-perturbative corrections, tend to lead to slowly varying potentials, without fine tuning, and typically large B-modes.

It is important to note that these models lead to large, typically super-Planckian field excursions. The Hubble scale being probed is well below the Planck energy, so the effective field theory is consistent, but it is obviously sensitive to the details of the UV completion. So it is of great importance to embed inflation within quantum gravity to obtain full control over these higher dimension operators in the effective potential. In other words, it is important to check if these simple symmetry arguments persist in the full quantum gravity theory, or if important modifications are present.

Observational data, including the possibility of a positive detection of B-modes, is very important to address these questions.

³ In other contexts, one can certainly introduce other sorts of fine tunings to obtain large amplitude B-modes also.

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Appendix A

In this appendix, we describe the predictions for the spectral index n_s and tensor-to-scalar ratio in simple single field models and then apply the analysis to the cosine potential of Section 6.2.

The spectral index n_s and the tensor-to-scalar ratio r are related to the slow-roll parameters ϵ and η by the following formulas

$$n_s = 1 - 6\epsilon_* + 2\eta_*, \quad r = 16\epsilon_* \quad (33)$$

The slow roll parameters were defined in terms of derivatives of the potential V in Eq. (2). The $*$ subscript here indicates that they need to be evaluated at the special moment when the modes leaves that we are interested in (namely those that affect the CMB). This is usually expressed in terms of the number of e-foldings of inflation N_e , which is given by

$$N_e = \frac{1}{M_{Pl}} \int_{\phi_e}^{\phi_*} \frac{d\phi}{\sqrt{2\epsilon(\phi)}} \quad (34)$$

(ϕ_e is the end of inflation).

In the case of the cosine potential given in Eq. (29), we find

$$\epsilon_* = \frac{M_{Pl}^2}{2F^2} \tan^2(\phi_*/2F) \quad (35)$$

$$\eta_* = -\frac{M_{Pl}^2}{F^2} \frac{\cos(\phi_*/F)}{1 + \cos(\phi_*/F)} \quad (36)$$

and the number of e-foldings is given by

$$N_e = \frac{2F^2}{M_{Pl}^2} \ln \left(\frac{\sin(\phi_e/2F)}{\sin(\phi_*/2F)} \right) \quad (37)$$

This allow a parametric representation of n_s and r as we vary the dimensionless quantity F/M_{Pl} for a given choice of N_e . For $F \gg M_{Pl}$ it is simple to show that this reproduces the predictions of $V \sim m^2 \phi^2$, including a near scale invariant spectrum. On the other hand, as we decrease F below M_{Pl} , the predictions deviate from scale invariance more and the tensor to scalar ratio decreases.

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